An imprecision importance measure for uncertainty representations interpreted as lower and upper probabilities, with special emphasis on possibility theory

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Uncertainty importance measures typically reflect the degree to which uncertainty about risk and reliability parameters at the component level influences uncertainty about parameters at the system level. The definition of these measures is typically founded on a Bayesian perspective where subjective probabilities are used to express epistemic uncertainty; hence, they do not reflect the effect of imprecision in probability assignments, as captured by alternative uncertainty representation frameworks such as imprecise probability, possibility theory and evidence theory. In the present paper, we define an imprecision importance measure to evaluate the effect of removing imprecision to the extent that a probabilistic representation of uncertainty remains, as well as to the extent that no epistemic uncertainty remains. Possibility theory is highlighted throughout the paper as an example of an uncertainty representation reflecting imprecision, and used in particular in two numerical examples which are included for illustration.

Key words: imprecision; importance measure; epistemic uncertainty

1 Introduction

The use of importance measures (IM) is an integral part of reliability and risk analysis. IM are used to study the effect on system level reliability or risk parameters of altering component level parameters. A number of uncertainty importance measures (UIM) have also been proposed in the literature (Aven &Nøkland, 2010). These extend the ‘classical’ reliability and risk IM in the presence of epistemic uncertainty. UIM are used to study to what degree uncertainty about risk and reliability parameters at the component level influences uncertainty about parameters at the system level.

In general terms, we are interested in the quantity Y, possibly a vector, and introduce a model g(X) which relates n input quantities $X = (X_1,X_2,\ldots,X_n)$ to the output quantity of interest Y. In particular,we are interested in an output quantity $Y= p = g(q)$, function of the input $X=q$ where p and q are reliability or risk parameters at the system and component level, respectively. Typically, p and q are given the interpretation of long-run frequencies, e.g. the fraction of time a system and its components are functioning, respectively. This is the interpretation adopted, for example, in the probability of frequency approach to risk analysis (Kaplan & Garrick, 1981).

Classical IM are used to analyze changes to p given changes to q. For example, the so-called ‘improvement potential’ of component i is defined as the change to the system availability p when the component availability q_i is set equal to 1, and the Birnbaum IM is defined as the partial derivative of p with respect to q_i(e.g. Aven & Jensen, 1999; Rausand&Høyland, 2004).

UIM are typically founded on a Bayesian perspective. A subjective probability distribution F is introduced for q and propagated through a model g. The result is a probability distribution of p, and UIMs are used to analyse changes to the distribution of p given changes to F. Reference is made to Section 2 for a brief review of IM and UIM.

In a Bayesian perspective subjective probabilities express epistemic uncertainty; hence, they do not reflect imprecision in probability assignments. The term imprecision here labels the phenomenon captured by a wide range of extensions of the classical theory of probability, including lower and upper pre-visions (Walley, 1991), belief and plausibility functions

(Dempster, 1967; Shafer, 1976), possibility measures (Dubois &Prade, 1988), fuzzy sets (Zadeh, 1965), robust Bayesian methods (Berger, 1984), p-boxes (Ferson et al., 2003) and interval probabilities (Weichselberger, 2000).

One much studied type of UIM is that reflecting the effect on system level parameter uncertainty of removing component level parameter uncertainty. For example, for a probability distribution F of component level parameters q which propagated through a model g induces a probability distribution of the system level parameter p, this type of UIM evaluates changes to the distribution of p by assuming q_i known for some i. Of course, the value of q_i cannot be specified with certainty and so the resulting measure becomes a function of q_i. An example is the measure Var(p) – Var(p|q_i), expressing the reduction in the variance of the system level parameter p that is achieved by specifying the value of the component level parameter q_i. One way to proceed is to consider the expected value of the above measure, namely (Iman, 1987):

$$Var(p) - E[Var(p|q_i)] = Var(E[p|q_i]). \tag{1}$$

Aven & Nøkland (2010) investigate the link between UIM and traditional IM. In doing so they distinguish between the cases that X and Y, as introduced above, are (a) observable events and quantities, such as the occurrence of a system failure and the number of system failures, and (b) unobservable parameters, such as p and q. Based on their findings a combined set of IM and UIM is defined.

Within a non-probabilistic framework, a Fuzzy Uncertainty Importance Measure (FUIM) has been proposed in (Suresh et al., 1996) to identify those component level parameters q_i having the greatest impact on the uncertainty of the system level parameter p. The FUIM measures the distance between the output fuzzy sets considering the input parameters q_i with or without uncertainty. In (Baraldi et al., 2009), the FUIM has been modified in order to consider the different imprecision in the output fuzzy sets, measured in terms of fuzzy specificity, instead of the difference between the fuzzy sets. In (Liping & Fuzheng, 2009), an importance measure based on the concept of possibilistic entropy has been proposed and applied to fault tree analysis in a possibilistic framework.

In the present paper, we consider the case that a distribution pair H_q is introduced for q. We may for example have $H_q = [N_q, \Pi_q]$, where N_q and Π_q are the cumulative necessity and possibility distributions (from possibility theory) of q, respectively; or $H_q = [Bel_q, Pl_q]$, where Bel_q and Pl_q are the cumulative belief and plausibility distributions (from evidence theory) of q, respectively; or $H_q = [H_q^l, H_q^u]$ where H_q^l and H_q^u are lower and upper imprecise probability distributions of q, respectively. In the present paper, possibility theory is highlighted throughout the paper as an example of an uncertainty representation reflecting imprecision. The choice of possibility theory in this early study of the suggested IIM is due to its mathematical simplicity; cf. Dubois (2006) who notes that 'Possibility theory is one of the current uncertainty theories devoted to the handling of incomplete information, more precisely it is the simplest one, mathematically'.

Defining the imprecision of a distribution pair as the area between its lower and upper cumulative distributions, we define an imprecision importance measure (IIM) that evaluates the effect on system level parameter imprecision of removing component level parameter imprecision. Two extents of imprecision removal are possible:

i. Removal of imprecision to the extent that a probabilistic representation remains
ii. Removal of imprecision to the extent that no epistemic uncertainty remains

The latter case may be seen as a special case of the former. The definition of an IIM in terms of imprecision removal is associated with an analogous problem as was seen above for uncertainty removal in the case of UIM; namely, the measure can be defined but neither the specific value of a component level parameter nor its probability distribution can really be specified. We are led to consider, respectively:

I. A probability distribution consistent with H_q
II. The IIM as a function of q_i

In the following we refer to these as type I and type II measures. Flage et al. (2011) study the type II measure. In the present paper,we extend the work of Flage et al. (2011) and study also the type I measure in the case that $H_q = [N_q, \Pi_q]$. A probability distribution is obtained from H_q by considering a possibility-probability transformation procedure, and further computations take place within the framework of a hybrid probabilistic/possibilistic method.

The remainder of the paper is organized as follows: In Section 2, we review some basic classical IM and some UIM. In Section 3, we review the concepts of uncertainty and imprecision, as well as their representation. In Section 4, we define an IIM as indicated above, and in Section 5 the suggested measure is evaluated in terms of a numerical example where possibility theory is used as the representation of uncertainty. Section 6 provides a discussion and some conclusions.

2 Importance measures and uncertainty importance measures

There are essentially two fundamental classical importance measures: the 'improvement potential' of a component, describing the effect on the system reliability of making the component perfectly reliable; the Birnbaum importance measure, reflecting the effect on system reliability of an incremental change in the reliability of a component. The improvement potential of a component is defined by (e.g. Aven & Jensen, 1999; Rausand & Høyland, 2004)

$$h(1_i, q) - h(q), \tag{2}$$

where h(q) is the system reliability function expressing p as a function of q; and $h(1_i,q) = h(q_1,...,1_i,...q_n)$ the system reliability function when component i is perfectly reliable. The importance measures referred to as risk achievement worth (RAW) and risk reduction worth (RRW) (e.g. Cheok et al., 1998; Rausand&Høyland, 2004; Zio, 2009) represent minor adjustments of the improvement potential importance measure. The Birnbaum importance measure is defined by (e.g. Aven & Jensen, 1999; Rausand & Høyland, 2004; Zio, 2009)

$$\frac{\partial h(q)}{\partial q_i}, \tag{3}$$

i.e. as the partial derivative of the system reliability with respect to q_i. The improvement potential importance measure is most relevant in the design phase of a system, whereas the

Birnbaum importance measure is most relevant in the operational phase (Aven& Jensen, 1999). See Rausand & Høyland (2004) and Zio (2009) for a more in-depth review of classical IMs.

Uncertainty importance measures were described to some extent in Section 1. The UIM by Iman (1987) is variance-based and hence an example of a measure in one of the three categories described by Borgonovo (2006):

i. Non parametric techniques (input-output correlation)
ii. Variance-based importance measures
iii. Moment-independent sensitivity indicators.

See Borgonovo (2006) for a more in-depth review of UIMs.

3 Uncertainty, imprecision and its representation

In engineering risk analysis a distinction is commonly made between aleatory (stochastic) and epistemic (knowledge-related) uncertainty (e.g. Apostolakis, 1990; Helton & Burmaster, 1996). Aleatory uncertainty refers to variation in populations. Epistemic uncertainty refers to lack of knowledge about phenomena and usually translates into uncertainty about the parameters of a model used to describe random variation. Whereas epistemic uncertainty can be reduced, aleatory uncertainty cannot and for this reason it is sometimes called irreducible uncertainty (Helton & Burmaster, 1996).

Traditionally, limiting relative frequency probabilities are used to describe aleatory uncertainty and subjective probabilities are used to describe epistemic uncertainty. However, as described in Section 1, several alternatives to probability as representation of epistemic uncertainty have been suggested, the motivation being to capture imprecision in subjective probability assignments. Imprecision here refers to inability to precisely specify a probability (distribution). Presumably an analyst/expert would ideally want to characterize epistemic uncertainty using a subjective probability (distribution); however, due to limitations in the information available (e.g. lack of data, lack of phenomenological understanding) the analyst/expert is unable or not willing to specify a single subjective probability (distribution) and only able to or willing to specify a probability interval (a family of probability distributions).

For example, numerical possibility distributions can encode special convex families of probability measures (Dubois, 2006). In possibility theory, uncertainty and imprecision is represented by a possibility function π. For each element ω in a set Ω, $\pi(\omega)$ expresses the degree of possibility of ω. Since one of the elements of Ω is the true value, it is assumed that $\pi(\omega) = 1$ for at least one element ω. The possibility measure of an event A, $\Pi(A)$, is defined by

$$\Pi(A) = \sup_{\omega \in \Omega} \pi(\omega), \tag{4}$$

and the necessity measure of A, N(A), by

$$\mathrm{N(A)} = 1 - \Pi(\overline{A}). \tag{5}$$

Uncertainty about the occurrence of an event A, then, is represented by the pair [N(A), Π(A)], where the necessity and possibility measures can be given the interpretation of lower and upper probabilities induced from specific convex sets of probability functions (Dubois, 2006):

$$\mathcal{P}(\pi) = \{P\colon \forall A \text{ measurable}, N(A) \le P(A)\} = \{P\colon \forall A \text{ measurable}, P(A) \le \Pi(A)\}. \tag{6}$$

Then, $\sup_{P \in \mathcal{P}(\pi)} P(A) = \Pi(A)$ and $\inf_{P \in \mathcal{P}(\pi)} P(A) = \mathrm{N}(A)$ (see e.g. Dubois & Prade, 1992).

Another point of view on possibility theory is a graded view where possibility measures express the extent to which an event is plausible, i.e. consistent with a possible state of the world, and necessity measures express the certainty of events. Reference is made to Dubois (2006) and the references therein.

4 An imprecision importance measure

Consider the system level reliability or risk parameter p and its distribution pair H_p induced by the propagation of the distribution pair H_q for a set of lower level parameters q through a model g. Define the imprecision of a distribution pair H, denoted ΔH, as the area between its lower and upper cumulative distributions, i.e.

$$\Delta H = \int (\max H(x) - \min H(x)) dx, \tag{7}$$

as illustrated in Figure 1.

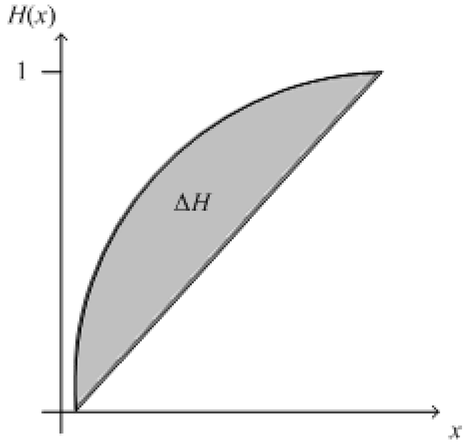

Figure 1.Imprecision measure ΔH equal to the area between the lower and upper distributions in the distribution pair H_p.

For example, in the case of a distribution pair H = [N, Π] induced by a triangular possibility distribution π with support S, we have – by geometrical considerations and recalling that a possibility distribution has unit height – that the imprecision of the possibility distribution is Δ(H) = |S| / 2. In the case of a probabilistic representation of uncertainty we have max H(x) = min H(x) for all x, and hence ΔH = 0.

Now define $\Delta_i(H_p)$ as the imprecision of H_p when the imprecision of the distribution on the parameter q_i is removed. We may then define an imprecision removal importance measure (IRIM) as

$$I_i = \Delta H_p - \Delta_{\mathrm{i}} H_p,$$

(8)

which expresses the amount of system level imprecision removal that comes from removing imprecision at the component level. The relative imprecision removal effect can be studied in terms of the measure

$$\bar{I}_i = \frac{\Delta H_p - \Delta_{\mathrm{i}} H_p}{\Delta H_p},$$

(9)

which expresses the fraction of imprecision associated with the distribution pair H_p that is attributable to component i.

As described in Section 1, imprecision can be removed either to the extent that a probabilistic representation remains, or to the extent that no epistemic uncertainty remains.

4.1 Type I measure

Removal of imprecision to the extent that a probabilistic representation remains means that uncertainty about q_i is described using a (subjective) probability distribution $F_i(x) = P(q_i \leq x)$, as illustrated in Figure 2.

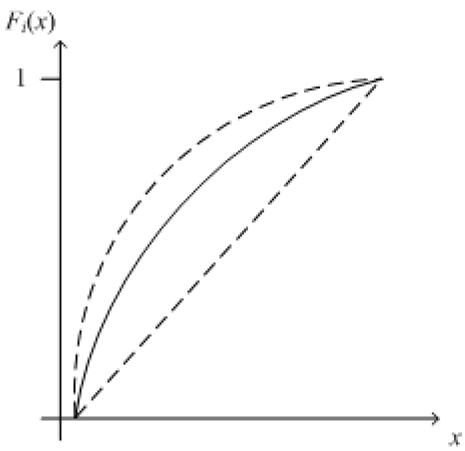

Figure 2.Removal of imprecision (imprecise probability distribution – dashed lines) to the extent that a single-valued probabilistic representation remains (solid line).

A probability distribution can be derived from a possibility distribution by considering a possibility-probability transformation procedure. Then a hybrid probabilistic/possibilistic method can be used for the joint propagation of the resulting probability distribution together with the remaining possibility distributions. In the following we review a possibility-probability transformation procedure based on Dubois et al. (1993) and a hybrid probabilistic/possibilistic method based on Baudrit et al. (2006) and applied in the context of risk analysis by (Baraldi&Zio, 2008).

4.1.1 Possibility-probability transformation procedure

Possibility-probability transformations (as well as probability-possibility transformations) are based on given principles and ensure a consistent transformation to the extent that there is no violation of the formal rules (definitions) connecting probability and possibility when possibility and necessity measures are taken as upper and lower probabilities, and so that the transformation is not arbitrary within the constraints of these rules. Nevertheless, as noted in (Dubois et al., 1993):

> ... going from a probabilistic representation to a possibilistic representation, some information is lost because we go from point-valued probabilities to interval-valued ones; the converse transformation adds information to some possibilistic incomplete knowledge. This additional information is always somewhat arbitrary.

When possibilityand necessity measures are interpreted as upper and lower probabilities, a possibility distribution π can be seen as inducingthe family $\mathcal{P}(\pi)$**P**defined in Equation (6) of probability measures. Since there is not a one-to-one relation between possibility and probability, a transformation from a possibility distribution π into a probability measure P can only ensure that

a) P is a member of $\mathcal{P}(\pi)$**P**
b) P is selected among the members of $\mathcal{P}(\pi)$**P**according to some principle (rationale); e.g. 'minimize the information content of P', in some sense

Various possibility-probability and probability-possibility transformations have been suggested in the literature.The principle of insufficient reason specifies that maximum uncertainty on an interval should be described by a uniform probability distribution on that interval. A sampling procedure for transforming a possibility distribution π into a probability distribution P according to the principle of insufficient reason is:

- Sample a random value α* in (0, 1] and consider the α-cut level $L_{\alpha*} = \{x : \pi(x) \geq \alpha*\}$
- Sample x* at random in $L_{\alpha*}$

The probability density f resulting from a transformation of π is given by

$$f(x) = \int_0^{\pi(x)} \frac{d\alpha}{|L_\alpha|}, \tag{10}$$

where $|L_\alpha|$ is the length of the alpha-cut levels of π. To motivate this, note that

$$f(x) = \int_0^1 f(x|\alpha) f(\alpha) d\alpha. \tag{11}$$

From step 1 in the sampling procedure above we have f(α) = 1, and from step 2 that

$$f(x|\alpha) = \frac{1}{|L_\alpha|}. \tag{12}$$

For the integration space we note that f(x|α) = 0 for α > π(x). The densityf is the centre of gravity of $\mathcal{P}(\pi)\mathbf{P}$. The transformation in Equation (10) applies to upper semi-continuous, unimodal possibility distributions π with bounded support.

Another possibility to probability transformation principle, based on maximum entropy, consists in selecting the P in $\mathcal{P}(\pi)$ which maximizes entropy. In general, however, this transformation violates the preference preservation constraint (Dubois et al., 1993) and is as such less attractive.

4.1.2 Hybrid combination procedure

By the hybrid procedure (Baudrit et al., 2006), propagation of uncertainty is based on a combination of the Monte Carlo technique (e.g. Kalos& Whitlock, 1986) and the extension principle of fuzzy set theory (e.g. Zadeh, 1965). The main steps of the procedure are:

- Repeated Monte Carlo samplings of the probabilistic quantities
- Fuzzy interval analysis of the possibilistic quantities

Considering the functional relationship p = g(q) studied in the present paper, the transformation procedure described in the preceding Section leads to a situation where uncertainty related to a single parameter q_i is described by a probability distribution F_i, while uncertainty related to the remaining n–1 parameters are described by possibility distributions ($\pi_1,\ldots,\pi_{i-1},\pi_{i+1},\ldots\pi_n$). For a fixed value of q_i, obtained by Monte Carlo sampling, the extension principle defines the possibility distribution of p as

$$\pi_p(x) = \sup_{q,g(q)=p} \min\{\pi_1, \ldots, \pi_{i-1}, \pi_{i+1}, \ldots, \pi_n\}. \tag{13}$$

We take m = 10^4 Monte Carlo samplings and determine the imprecision reduction from the transformation from π_i to F_i as the average imprecision reduction.

4.2 Type II measure

Removal of imprecision to the extent that no epistemic uncertainty remains means that q_i can be specified with certainty, and the uncertainty hence represented by the step function $u_i(x)$, where $u_i(x)$ is equal to 0 for $x < q_i$ and equal to1 for $x \geq q_i$, as illustrated in Figure 3.

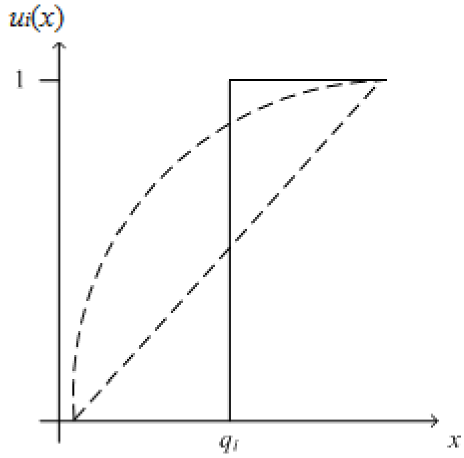

Figure 3.Removal of imprecision (imprecise probability distribution – dashed lines) to the extent that a no epistemic uncertainty remains (solid line).

In the case of removal of imprecision to the extent that no imprecision remains, we are led to consider the suggested imprecision importance measure as a function of q_i, denoted $I_i^{II}(q_i)$.

Section 5 presents a numerical example evaluating type I and type II measures.

4.3 Imprecision importance measures in presence of dependences

Future work will be devoted to the investigation of the proposed imprecision uncertainty importance measures in presence of dependences in the input considered for the analysis. In practice, two types of dependencies may need to be considered: i) epistemic dependence between the uncertainty on the component parameters and ii) physical dependence between the system components. The former case relates to situations in which the information on the values of the parameters of two or more system components is correlated. For example, if there are two identical components in the system and the same information is used to estimate their characteristic parameters, then the uncertainty on them will be the same and identically represented. In this case, the procedures of uncertainty removal should be modified in order to consider that the reduction of the uncertainty on a single component parameter can cause the (same) reduction of the uncertainty on other correlated parameters. Contrarily, the physical dependence between the system components is not expected to influence the procedures of uncertainty removal, since this dependence has an effect on the aleatory character of the modeled process but not on the epistemic uncertainty on the component parameters.

On the contrary, the procedure for the propagation of the uncertainty from the component level parameters (input quantities) to the system level parameter (output quantity) should be modified in both cases of dependence. On this subject, the interested reader may refer to Pedroni and Zio (2012).

5 Numerical example

Consider a system S consisting of five independent components connected asillustrated by the reliability block diagram in Figure4.

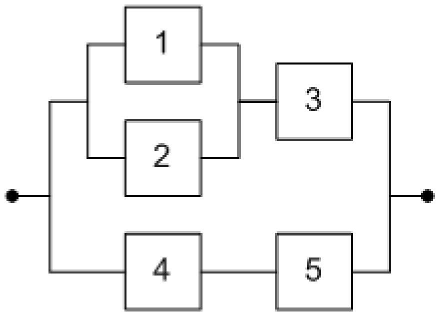

Figure 4. Reliability block diagram of system S.

Component i has availability q_i, i = 1, 2, 3. The availability of the system, denoted p, can then be expressed as

$$p = 1 - \big(1 - \big(1 - (1 - q_1)(1 - q_2)\big)q_3\big)(1 - q_4 q_5). \tag{14}$$

The component availability parameters $q = (q_1,q_2,q_3,q_4,q_5)$ are assumed to be unknown, the uncertainty being described using marginal necessity/possibility distribution pairs $H = (H_1,H_2,H_3,H_4,H_5)$, where $H_i(x) = [N(q_i \leq x), \Pi(q_i \leq x)]$, i = 1, 2, 3, 4, 5.Due to the restrictions in terms of the type of possibility distributions that the transformation method described in Section 4.1.1 applies to, only triangular possibility distributions will be considered in relation to the type I measure in Section 5.1. In Section 5.2 also trapezoidal and uniform possibility distributions are considered in relation to the type II measure.

5.1 Type I measure

We assume that the distributions on the component availabilities and the resulting distribution on the system availability are asshown in Figure 5.

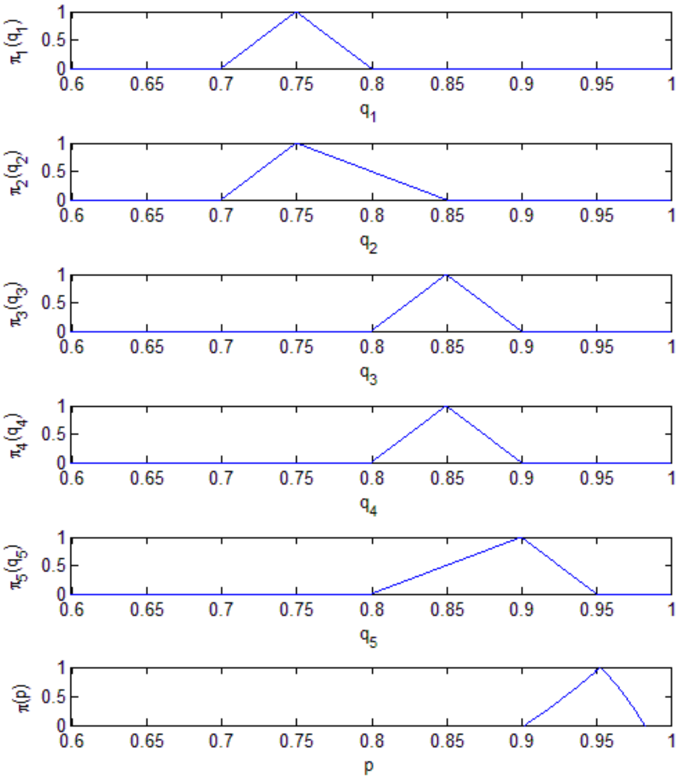

Figure 5. Input distribution functions on component availabilities and resulting system availability.

Let s_1, $s_2$and c denote the lower support limit,the upper support limit and the core value of a triangular possibility distribution, respectively. For this type of distribution the imprecision equals

$$\Delta H = \frac{s_2 - s_1}{2}. \tag{15}$$

Table 1 lists the possibility distribution parameters and the associated imprecision at both component and system level. The imprecision related to the resulting distribution for the system availability is determined as the Riemann sum over c = 10^3 α-cuts.

Table 1.Component and system availability distribution parameters and imprecision for system S.

i	s_1	c	s_2	ΔH
1	0.70	0.75	0.80	0.050
2	0.70	0.75	0.85	0.075
3	0.80	0.85	0.90	0.050
4	0.80	0.85	0.90	0.050
5	0.80	0.90	0.95	0.075
System	0.90	0.95	0.98	0.040

Table 2 summarises the values of the type I imprecision importance measure. The imprecision importance ranking is [5, 3, 4, 2, 1].

Table 2.Type I imprecision importance value ranges.

i	I_i^I	$\bar{I}_i^I$
1	0.0026	6.48 %
2	0.0039	9.89 %
3	0.0116	29.1 %
4	0.0087	22.0 %
5	0.0129	32.4 %

Notice that although components 2 and 5 are characterized by the same imprecision ΔH = 0.075, component 2 imprecision importance measure is lower than that of component 5 due to their different position in the system block diagram. In particular, since component 2 is in parallel to component 1, a failure of component 2 does not cause the unavailability of the upper system branch. Thus, component 2 has a lower impact on the system unavailability imprecision than component 5 whose failure would cause the unavailability of the bottom system branch.

5.2 Type II measure

We now assume that the distributions on thecomponent availabilities and the resulting distribution on the system availability are asshownin Figure 7.

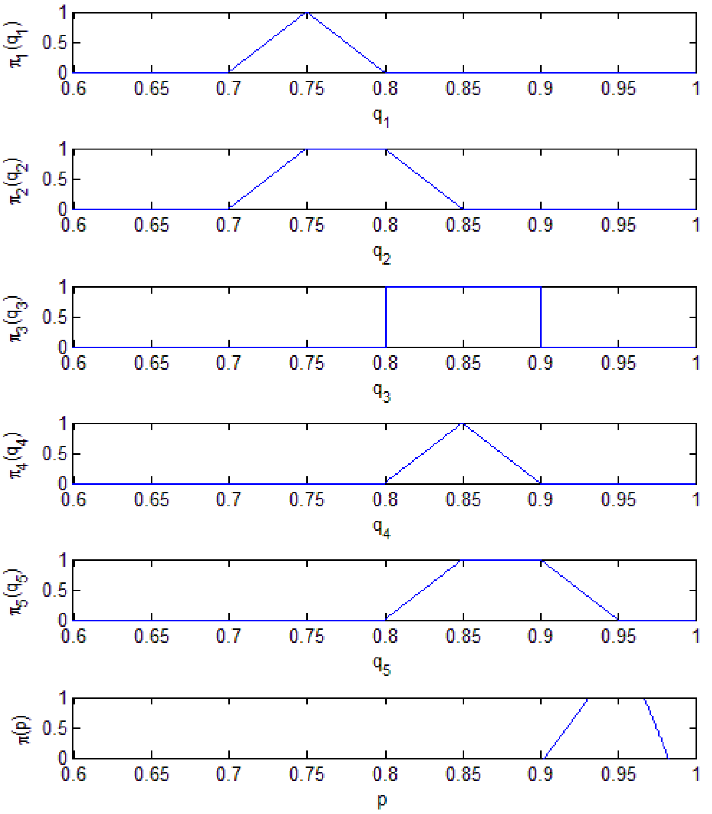

Figure 7. Input distribution functions on component availabilities and resulting system availability.

Let s_1 and s_2 (c_1 and c_2) denote the lower and upper support (core) limits of a possibility distribution, respectively. For a trapezoidal distribution we have $s_1 < c_1 < c_2 < s_2$, for a triangular distribution $s_1 < c_1 = c_2 < s_2$, and for a uniform distribution $s_1 = c_1 < c_2 = s_2$. For these distribution classes we then have that the imprecision equals

$$\Delta H = \frac{s_2 - s_1 + c_2 - c_1}{2}. \tag{16}$$

Table 1 lists the possibility distribution parameters and the associated imprecision, at both component and system level.

Table 3.Component and system availability distribution parameters and imprecision for system S.

i	s_1	c_1	c_2	s_2	ΔH_q
1	0.70	0.75	0.75	0.80	0.05
2	0.70	0.75	0.80	0.85	0.10
3	0.80	0.80	0.90	0.90	0.10
4	0.80	0.85	0.85	0.90	0.05
5	0.80	0.85	0.90	0.95	0.10
System	0.90	0.93	0.97	0.98	0.057

Figure 8shows the relative variant of the type II imprecision importance measure for all five components in system S.

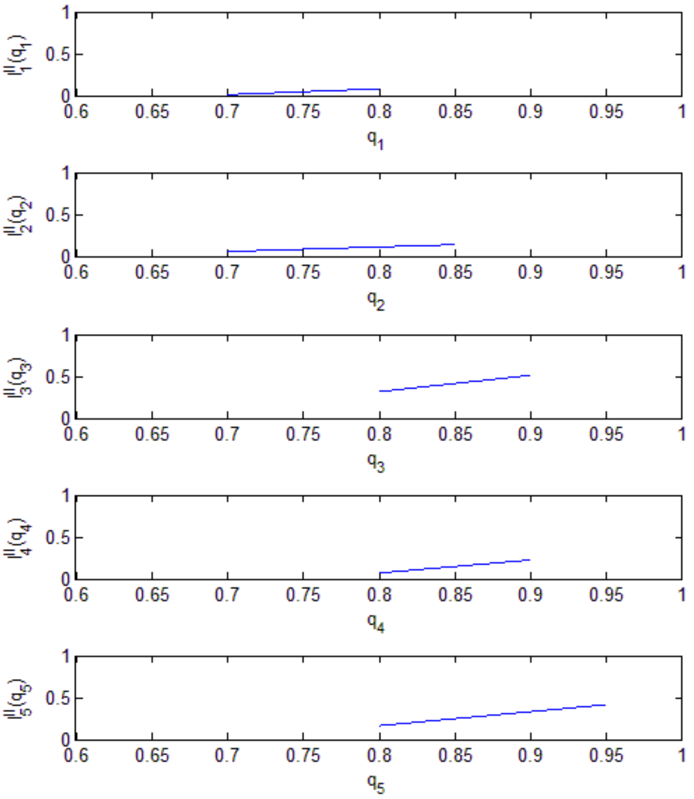

Figure 8.Relative variant of the type II imprecision importance measurefor each component insystem S.

The (relative) imprecision importance for each component is evaluated as a function of q_i on the support of the associated distribution. Table 2 summarises the value ranges of the (R)IRIM. The imprecision importance ranking is [3, 5, 4, 2, 1] according to both high and low values.

Table 4.Type II imprecision importance value ranges.

i	I_i^{II}	$\bar{I}_i^{II}$
1	[0.0006, 0.0046]	[1.13 %, 8.10 %]
2	[0.0032, 0.0079]	[5.60 %, 13.8 %]
3	[0.0182, 0.0294]	[31.7 %, 51.2 %]
4	[0.0040, 0.0129]	[6.94 %, 22.5 %]
5	[0.0094, 0.0239]	[16.4 %, 41.6 %]

Notice that, although components 1 and 2 are in parallel, component 2 is characterized by larger Type II imprecision importance value ranges than component 1. This is due to the fact that our knowledge on q_2 is more imprecise than that on q_1, being $\Delta H_2 = 0.10$ whereas $\Delta H_1 = 0.05$. Thus, as expected, removing the imprecision on the more imprecise input causes a larger reduction of the system unavailability imprecision.

6 Discussion and conclusions

In the present paper, we have described and applied an importance measure that can be used to evaluate the effect on system level parameter imprecision of removing component level parameter imprecision. Hence, the suggested measure is defined analogously to the classical improvement potential IM which describes the effect of removing the unreliability of a component, and analogously with a number of UIMs that describe the effect of removing uncertainty about component performance.

Two extents of imprecision removal are considered: reduction to a probabilistic representation (type I) and removal of epistemic uncertainty (type II), the latter a special case of the former.

The relative version of the measure expresses the fraction of the initial amount of imprecision on the system level parameter that is attributable to each component. In a ranking setting this format is perhaps easier to comprehend than the underlying absolute numbers; however, the fractions need to be seen in relation to the initial amount of imprecision on the system level parameter.

IIMs may be seen as an extension of UIMs when the uncertainty representation is no longer single-valued probability but instead some alternative representation with the interpretation of lower and upper probabilities.

An alternative to the measure ofimprecision used in the present paper, and hence relevant for future work,is the Hartley-like measure of non-specificity, variants of which exist for both possibility and evidence theory; see Klir (2006; 1999).

Further work in relation to the suggested measure could also be directed towards implementation of the type II measure on more complex systems. Moreover, possibility theory provides a relatively simple and hence convenient uncertainty representation to use for the implementation of the suggested measures; however, other representations couldalso be considered in terms of application depending on the particular uncertainty setting. Finally, future work will also be devoted to the investigation of the proposed imprecision uncertainty importance measures in presence of dependences in the input considered for the analysis.

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